Mathematics and Religion
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The Philoctetes Center

Levy:	**Francis Levy**
Nersessian:	**Edward Nersessian**
Balestra:	**Dominic Balestra**
Goldstein:	**Rebecca Newberger Goldstein**
Graham:	**Loren Graham**
Nelson:	**Edward Nelson**
Tegmark:	**Max Tegmark**
Audience:	**Question from audience**

Levy: I'm Francis Levy. Ed Nersessian and I are co-directors of the Philoctetes Center, and welcome to *Mathematics and Religion*. Before we begin today's program I wanted to again call your attention to the exhibit on the walls, which is the *Aesthetics of Mathematics*, which is curated by Hallie Cohen and Adam Ludwig.

Now I'm pleased to present Rebecca Newberger Goldstein. Rebecca Newberger Goldstein is a novelist and philosopher. She is the author of eight books, six of them fiction, including the best-selling *The Mind-Body Problem*. Her last two books have been non-fiction: *Incompleteness: The Proof and Paradox of Kurt Gödel*, which was chosen by *Discover Magazine, The Chicago Tribune*, and *The New York Sun* as one of the best books of 2005, and *Betraying Spinoza: The Renegade Jew Who Gave Us Modernity*, which won the 2006 Koret International Award for Jewish Thought. The recipient of numerous awards for scholarships and fiction, she received a MacArthur "genius" award in 1996—and I didn't need to know she'd gotten a MacArthur to know that she's a genius, because we were talking on the way over here. At a certain point in the conversation I was with her, and then at another point I knew it was making a lot of sense but that I couldn't follow it any longer. [laughs] It was the part about your PhD thesis on hard consciousness. And in recognition of her talent—this is what the award was for—she got the award for "dramatizing the concerns of philosophy without sacrificing the demands of imaginative storytelling." Her newest book is entitled *Thirty-Six Arguments for the Existence of God: A Work of Fiction* [laughter]—there, you see what I mean [laughs]—and will be published by Pantheon in 2009. She is a research associate at Harvard University. Dr. Goldstein will moderate this afternoon's panel and introduce our other distinguished guests.

Goldstein: Thank you so much for inviting me and for organizing such wonderful meetings of the minds, and may you continue your amazing work.

Levy: Thank you.

Goldstein: It's my pleasure to introduce this distinguished mind-gathering. And, to my right is Professor Dominic Balestra, who is a philosopher, former Chair of the Philosophy Department at Fordham, also the former Dean of the Arts and Sciences faculty at Fordham University, and he is

the author of many articles, he has given many talks, many of them directed toward the juncture between science and religion, philosophy and religion—I hope that is a fair description.

And to his right is Max Tegmark, who is a physicist and a metaphysician I would say [laughs] as well. He is Associate Professor at MIT. He's previously taught at the University of Pennsylvania, served as a Hubble Fellow, and he is somebody who I have been trying very hard to talk into writing a book, because I think his ideas, as you are going to see today, really deserve a full-length book.

It is my pleasure to introduce Professor Loren Graham, who is Professor of the History of Science Emeritus at MIT, currently an associate researcher at Harvard University, and those of you who were here this morning had the pleasure of hearing him give a talk on his latest book, *Naming Infinity*. I'm sure we're going to be talking about—telling a fascinating story of the intersection between mysticism and mathematical creativity.

To my left is Professor Edward Nelson, a very distinguished mathematician at Princeton, winner of the American Mathematical Society Steele Prize for seminal contributions to research, and I only recently learned from what I was given, has spoken at Vatican, giving a talk on mathematics and faith.

So certainly we have here a novelist and a philosopher, a philosopher, physicists, historian of science and a mathematician. And we're going to talk about not science and religion—that has been spoken about so much recently. We're going to talk about mathematics and religion, which has not received as much attention I think. And we want to see if there's anything that these two fields, disciplines, questions, sets of questions have in common. And I thought I would just set the stage by talking a little bit about why would you think that they have anything in common. It's much more natural to think they've got nothing in common, mathematics and religion. Mathematics is our clearest domain of human knowledge. It's the one place where we have certainty—it would seem. That's the way we think about it. It's the one place where we have proofs. And these proofs, when you prove something in mathematics it's immune to revision. It doesn't matter what empirically—unlike physics or cosmology, it doesn't matter what we empirically observe of the world. We are never going to discover that five plus seven doesn't equal twelve. If I count five things and seven things and I come up with thirteen things I recount. And if I keep getting thirteen I think, well, some kind of splitting has taken place. Or I'm dreaming. Or even, I'm going mad. But no experience is going to cause me to take back the belief that five plus seven equals twelve, and much more interesting mathematical results as well. Nothing that we could observe—it's certain, it's immune, and we know it with great clarity and penetration.

Unlike our knowledge of God, if there is such a thing, or beliefs in God. If ever there is a place where there's uncertainty and people have intuitions, and their intuitions can't possibly be right because people with very certain intuitions disagree with one another, and *kill* each other because of their *very* certain intuitions, if there's one place where we think we know you can't trust intuitions it's on the subject of God and religion. So why would one even think that mathematics and religion have anything to do with one another? They feel so different it seems on the surface.

Mathematics however is very, very certain, but there are a lot of questions about mathematics, what we call metamathematical questions, which are completely uncertain, including what is it that we're doing when we do mathematics. What is it that we're knowing. Are we discovering, are we creating, are we making up the rules as we go along, is it a higher form of chess? Or is it some kind of real objective discovery—and that means if it's not discovery of this world, because although it describes this world it can't be empirically invalidated by this world, is it of another world, of a supersensible world, a transcendent world? And so you see how these questions about mathematics and questions about supersensible, transcendent God might possibly have something to do with one another.

Also, mathematics often, although it perceives by proofs, proofs are often the afterthought. It's after you've already seen it, you have a heuristic grasp of it. Again, if this is a transcendent world, what are we seeing when we're seeing these things, and how are we making contact with this transcendent world? So there is—although mathematics itself seems very, very certain, mathematics raises all sorts of questions about human knowledge and the limits of human knowledge, and how we can do what we do when we're doing mathematics.

So that is a little bit of, to cede a little bit of uncertainty about mathematics and our knowledge of mathematics that might have something or other to do with issues of God—certainly have been claimed to. So that's a little stage setting for why mathematics and religion might come together in a discussion. And what I would like to do now is, I guess—Loren Graham was very, very honest this morning, and he kind of just laid it on the table what his view is of the God issue, and also, I guess maybe even a little bit of mathematical truth, so it would be interesting I thought for all of us to go around, to start off the discussion, and say, first of all, if you have a view about mathematical truth, do you think that it's objective and independent, or kind of we make up our formal systems and we crank out the results, and what your view is on God, and do you think that there is in fact any intersection between your two views. Are you influenced in any way one way or the other on these two issues?

So, Dominic?

Balestra: Wow, that's a large order. My views on mathematics and my views on God, wow.

Goldstein: And then there'll be no secrets from any of us.

Balestra: Well, on the mathematics thing, question, area, you know, there are three—you might say, there's a little more now. At the turn of the twentieth century, entering it, there were like three schools of thought: the Platonist, where you believe there are these mathematical entities that are nonphysical, non-empirical—eternal objects, if you will—and the mathematician, when the mathematician comes to know them, or know truths about them, he or she is dealing with this world that transcends in some sense the physical world. And that goes back to Plato, who said it so well.

Then, another approach of formalists, who reject that completely and want to say all mathematics is about is syntax, these symbols that they're manipulating, and there are rules of syntax—it's about logical proof, conjecture, a theorem, and then you logically prove it. Usually you work

from a system of axioms, like in Euclid's geometry, and then you prove theorems that follow using well established logical rules of inference.

Now, what's interesting is—and now I'm going to maybe pose a question to what you said—what's interesting though is in mathematics we know now you can prove theorems in geometry that contradict what you can prove in Euclidian geometry—we know that non-Euclidian geometries emerged. Which raises interesting questions about the creativity question, and maybe that's what part of the meaning of the freedom of the mathematician, what Gödel talked about is there is a certain freedom there, a certain freedom, a certain play where if you can make that right move, it opens up a whole new world. So those mathematicians who were working on—what they were trying to do was an indirect proof of the fifth postulate of Euclid, the parallel postulate, and they couldn't derive a contradiction by negating it and joining it with the others, so they started generating all these interesting results, and I guess someone finally realized, hey, you know, this is an alternative geometry. It's opened up a whole other, you might say, world of space.

And so you say now, are they in contradiction? Well, yes and no. And I'm going on and on here, but—so, the formalists could maybe play with that and have some fun. You have the Platonists, and then there's that third view, I guess they were called the intuitionists, which is a strange name for them, but it really comes from Kant, the philosopher, and it had to do with Kant's view of mathematics—which was I think another word for the intuitionist's view of math is the mathematical entity that's known by the mathematician is constructed in the mind of the mathematician, and it's constructivism. But what is it, what's the material of the construction? What is it that the mathematician is constructing, shaping, forming? Now, for Kant it had to do with the forms of—very transcendental theory of Kant, but it had to do with an idealism, the forms of space and time internally to the mind, the human mind, something like that.

So those are three views. I have to confess, I lean toward Plato, partly because of Gödel's proof. He showed that there's more mathematics than we can prove, something like that; that's very crudely said. But also I think I lean that way because at the end of the day Plato is my philosopher. And, you know, a remark was made in response to the lecture this morning about—by Whitehead, but it really goes back to Plato: is this mathematics that we do a kind of divine madness? I'm not a Platonic scholar. I don't know Plato that well, but I know him well enough to hazard this: what you find in Plato is he pushes the boundaries of reason, rationality, intelligence as we understand it to where it can no longer answer the question, and he characteristically when he pushes it there turns to myth. And it's the mythic where the divine, if you will, resides. And this capacity that humans have to do mathematics in such a creative way, Plato's suggesting it is something of a divine gift. We can't account for it and explain how it comes about, but we have it, and that's the divine element in us.

Now, what I'd like to do, but I'll stop here—maybe in the discussion—I'd like to connect that with Spinoza. And there's something like that that runs in Spinoza. Now, it won't give you a personal God, I don't think. Is it compatible with religions that have a more personal God? I think that's a very large question, and I don't know enough sophisticated theology to puzzle that out—because theology when it's done well is sophisticated and very nuanced, and not easy.

And I still haven't come clean on, am I a theist? Yes. I'm a practicing Catholic—probably a heretical one, if you really ask me all the things I believe. But that's where I come from.

Tegmark: So Rebecca, you asked us to start off by talking about our own views or guesses, both what's going on with mathematics and with religion. To me these two questions are really the same questions. You're asking what do we believe about the ultimate nature of reality, and this is a basic question which really gets me fired up in my life, and I feel so lucky to have a job where I get to spend a lot of my time thinking about the ultimate nature of reality, and that they even pay me for it is a scam. I hope my department chair isn't watching this on webcam.

In terms of the three views which you nicely summarized there, Platonists, formalists and intuitionists, I'm very much a Platonist, and perhaps the most extreme Platonist that I've come across. So extreme in fact that I think I belong to a minority of one among my physics colleagues at least. I have this very heretical view that physical reality and mathematical reality are actually one and the same. If you go back in history, already Galileo exclaimed that nature is a book written in the language in the mathematics. And he didn't explain why, but he noted that many of the irregularities can be captured very nicely in this language. And since then we've seen there's more and more mathematical irregularities cropping up around us, but after relativity theory and quantum mechanics had come along, which are very mathematical, Eugene Wigner wrote this beautiful essay in the '60s, *The Unreasonable Effectiveness of Mathematics in the Natural Sciences*, where he felt that this was such a striking success that it really called out for some kind of explanation. He again didn't really give one, but—anyway, my guess is that the explanation is that reality is so well described by mathematics because it is ultimately purely mathematical, and not only is our physical world described by a mathematical structure, but it is a mathematical structure. So I think we're all living in a gigantic mathematical object—not one of the simple ones that we learn about in high school math. We're not living inside of a cube or a dodecahedron or in the set of integers, but there's some more complicated mathematical object, maybe M-theory, maybe some—more likely something we haven't discovered yet which somehow is our reality.

This is my personal guess as to what's going on. But I try to keep a very open mind. I think humility is very much in order when speculating about these big questions, and I think if I had to summarize everything I feel I've learned that I'm really convinced about so far from my life in science, I think I would summarize it by saying that I'm really convinced that the ultimate nature of reality, whatever it is, is very different from the way it first seems.

Graham: Well, as I said this morning, I am a secularist. That is to say I am not religious. However, I just wrote a book which is about how religion helped mathematics in a particular instance. As a secularist I'm not too persuaded by most arguments linking science and religion. I find arguments coming out of physics and biology and chemistry linking science to religion weak and contingent. I'd like to think that I'm tolerant enough and big enough to hear those arguments and to recognize that very important people have been moved by them. I don't dismiss them. I'm just saying that I do not find them personally very persuasive.

But I would sort of go almost against something that Rebecca said—although she didn't *assert* it, she just portrayed this—and say that all the arguments linking science, if you want to include mathematics, to religion, the ones that I find the most interesting—I'm not going to quite say

persuasive, but the ones I find the most interesting are in mathematics. There are non-trivial arguments in my opinion in mathematics linking it to religious concepts. I think it has to do with the fact that, as Hermann Weyl said, some mathematicians say that mathematics is the study of infinity. And once you start thinking about infinity, the supposed links to religion get rather obvious. God is supposed to be of infinite quality, omnipotent and so forth. Many of the adjectives that one uses to explain divinity are strikingly similar to adjectives that we use to explain mathematical infinity, and many people have been moved by this. And some of them are important mathematicians: Nicholas of Cusa, Hermann Weyl, Arthur Stanley Eddington, and my Russian, if I can say that, Russian mathematicians, Nikolas Luzin and Dmitri Egorov. The arguments that they use to talk about infinity, and types of infinity, and names for God and names for divinity, are in my opinion not trivial. They're worthy of consideration. I try to lay them out in my book—one of my graduate students heard about the book and came up and he said, "Loren, you're not going mystical on me, are you?" And there are people who think that about anyone who gave a sympathetic account, as I hope I gave, to these religious mystics and their work on mathematics. If you think that means I'm sort of halfway in their camp I would say no, but I appreciate them, I understand them, I'm thrilled by them, I share their aspirations. I'm not in the end quite persuaded.

Georg Cantor taught us that there are different types of infinities, even in mathematics: countable infinities, non-countable infinities and lots of other kinds of infinities. Well, just like there are different kinds of infinities in mathematics, in my opinion there are different kinds of infinities outside of mathematics, and therefore it isn't obvious to me that when we speak of God or divinity, in terms of omnipotence or other infinite qualities, that that infinity is the same as infinity in mathematics, and therefore they can be equated or linked or made a strong bond. I have serious doubts about that. But I still would like to pay a compliment to these distinguished mathematicians who have made the argument that they have a mystical intuition that is linked to their belief in God that helps them when they do mathematical work. That can't be dismissed. It's a historical fact.

Nelson: Well, we have a wide spectrum with this panel. I'm an extreme formalist, and the interesting thing about the three schools of thought about the foundations of mathematics is that it doesn't make any difference when it comes to the practice of mathematics. Mathematicians who think about foundations might describe themselves with one of these three terms, but we all agree as to whether mathematics is correct or not after it's been sufficiently studied. And this is really remarkable when you think about it. What other discipline, what other field of human endeavor has this extraordinary consensus over time and space as to whether something is correctly proved or not?

Tegmark: Physics. You have people with very different metaphysical beliefs about what's going on, but if the experiment shows that this theory is wrong—

Nelson: Well, once you come to the experiment, but then in terms of theory, I mean the divergent views on whether string theory is correct physics or not, that doesn't happen in mathematics.

Goldstein: I mean you have these people who have extremely different interpretations of what's going on in quantum mechanics, but they all work together and apply the theory, so—would you accept that that happens in physics?

Nelson: No, I think there's a qualitative difference in mathematics, that the consensus is truly remarkable and unique. That's my own feeling.

Tegmark: But on a positive note, I think this diversity in philosophical views that we see both within physics and math are really a strength rather than a weakness of our communities, because if we had some kind of Microsoft monoculture in terms of ontology where everybody believed the same thing, that would mean that everybody was excited about doing the same kind of research—

Nelson: I agree.

Tegmark: And everybody would be looking under the same lamppost, and precisely because different people are passionate about doing very different things we get so much more diversity and we find some things which really work out, and it makes us much stronger as a community. So I never feel compelled to try to convince someone who believes other than myself to think my way, because I think it's much healthier to have this range of views.

Nelson: Another interesting feature of mathematics is the tremendous timeframe. In ancient Greece people worried about, or thought about the nature of perfect numbers. Six is called a perfect number because it's the sum of its divisors other than itself; one and two and three divide six, one plus two plus three equals six. And Euclid not only did geometry but did number theory in his book, and he proved that numbers of a certain form connected with prime numbers were perfect. And so, yes, it was 2000 years later, in the eighteenth century Euler proved that every even perfect number was of Euclid's form. That's a tremendous timeframe to be working on the same problem, and it's a major open problem today, do there exist odd perfect numbers. No one knows.

Now, I described myself as an extreme formalist. That means that David Hilbert, who is regarded as the founder of formalism, I think was in his heart a Platonist. [laughter] I believe he adopted formalism as a strategy against his antagonist, the intuitionist, L.E.J. Brouwer. For example, he set the program of proving that mathematics was consistent, starting with number theory, to prove that number theory was consistent. Well, that's a mistake. The question is: *is* elementary number theory consistent or not? I believe it's not. I believe that we do not have certainty in mathematics. I believe that many of the things we regard as being established in mathematics will be overthrown. And I'm a crackpot in that respect.

Tegmark: Just to clarify, do you believe that one day someone might actually prove within number theory axioms that zero equals one, so that everything would just come crashing down?

Nelson: Yes, that's what I'm working on.

Levy: What does that mean you're working on it, just for a layperson? How do you work on it?

Nelson: That I'm keeping to myself for the time being. Give me two years.

Tegmark: I just can't wait to see Rebecca's expression when you prove for her that therefore five plus seven equals thirteen—

Nelson: Well that means there's something wrong with the axiom, not that there's something wrong with seven plus five—

Goldstein: Exactly.

Nelson: But as I said before, mathematicians of all schools of thought agree as to whether something is deduced correctly from the axioms. The question is whether the axioms are consistent or not.

In terms of religion, I'm a Christian. Worship and prayer are very important to me. I experience religion more as a matter of faith than of a belief system, and I think there's one question where mathematics and monotheistic faith seemingly conflict, and that is in the nature of mathematical objects. Let's take the numbers, 1,2,3,4,5 and so forth—0 too. But also more general mathematical objects, like exotic spheres and whatnot. Do these things exist? Well, as a formalist I believe that we make them. But if we believe that the numbers, 0,1,2,3,4 exist as a completed infinity, where did they come from? Have they existed as they were in the beginning or now and ever shall be? That sounds like a religious belief—not mine.

If we believe they're created, that raises a problem, because I believe it's part of monotheistic faith that all of creation is contingent; it could have been created otherwise. How could the numbers have been created otherwise? I think there's a real problem here.

Goldstein: Okay, so I'm going to spill my guts too. I'm a reluctant, or a skeptical Platonist. I wish I weren't a Platonist. It's too mystical for me, I don't like it, but I think—for example there's a famous unsolved problem, Goldbach's Conjecture, every even number is the sum of two prime numbers, hasn't been proved false. If it is false then in principle one can show it. One just has to keep going far enough and you'll come to the counterexample. If it's true we may never know, there may never be a proof that gets us to it, but I think it has an answer. I think Goldbach's Conjecture is either right or wrong, every even number is the sum of two prime numbers or it isn't. That makes me a Platonist. And I can't make my way around it—I also found, I started studying Gödel's Incompleteness Theorems when I was a graduate student hoping that it wouldn't somehow seem to point the way to Platonism, as he thought it did, and I think it does. And I don't like it. That's not my orientation, but until somebody—perhaps you [laughs]—can show me how we dance around, how we get around Gödel's Incompleteness Theorems I'm going to feel—I'm going to have to give it some credence, Platonism.

And I am an atheist. I am not wishy-washy on this question. Not only do I think the arguments for God's existence don't work, I think that this, more importantly to me, does not look like the kind of world empirically that is created by a good and caring and powerful God. It just—to me there's just too much empirical evidence against it. Suffering of children is my number one complaint. And the amount of work that one has to do, that philosophers have done, that theists have done to answer the question, the problem of evil—you know, free will, and that works for only some of them, and the Holocaust was, okay, the Nazis had to have the power of absolute evil in order for them to be free, so a certain amount of suffering had to take place—that even

that only goes so far. There's a lot of suffering that can't be answered that way. Soul making, you know, this is a place where a lot of virtues can only be induced, we can only come to them because of suffering, that doesn't really seem to be to explain the suffering of children.

Anyway, to me this just doesn't look like a world created by God, and so I am an atheist. I mean I don't—I'm not an agnostic, I'm an atheist.

Levy: Why must God be a producer of only good things—your definition seems to presuppose the notion of goodness—

Goldstein: Well, yes, I mean it's absolutely right, so the God that I would want to sort of pray to and care about and that sort of thing, I don't see this as that kind of God—any other thing you want to call God. I've written a book on Spinoza, and I am I suppose—it depends what you mean by Spinozist, but I would define myself as a Spinozist. I do believe in Spinoza's notion of God, which is a non-transcendent God, the world itself is God, understanding the world—

Balestra: That's not Spinoza.

Goldstein: Well—

Balestra: I think that's a mistaken reading of Spinoza.

Goldstein: Well, we can argue about that. We can—

Balestra: I can give you the place in book one of *The Ethics* where it stands out—

Goldstein: Tell me.

Balestra: What he says there, he talks about God, and then let me just—and I want to link Spinoza, it's not unrelated I think to what you said about zero and one. Very broadly: Spinoza, Plato, *The Parmenides*. Parmenides, some of you may know, some of you may not, pre-Socratic, early—at the early kind of dawning of what we call philosophy in the west is this Parmenides, this thinker. There's Thales and others, but Parmenides to me is the one who nails it, because he's the one who starts separating what we think of, what emerges as rational thinking as distinct from mythic-like thinking, or thinking that has it all undifferentiated, zero and the ones. And then you start seeing it differentiating. And Parmenides then reflects on this, that great fragment we have from him, but basically says, look, ultimately anything—he refers in an indefinite pronoun to *it* is or it is not, and he says if it's the 'it is,' it is and cannot not be. In other words—and then he says and 'it is not' cannot be and is unthinkable. And then he stresses this—now, there are critiques later in philosophy that he's confusing the predicative use of the 'is' with the identity use and the assertive 'to be' use and all that, but when you come clean on it—and this continues in philosophy—what he's saying is the question of being kind of gradually emerges. Think 'it is' is, or it is not.

And then he suggests, and this is what the tough challenge is, it is must necessarily be, and it is not cannot be, and it's not. Then he says there's another alternative that many think is thinkable, and he's saying this is a way that's confusing, it's mere opinion and false: that it *is* and *is not*. And that's what the whole, all the particularity of the world is about. I am, but I'm not *you*. You

are, but you're not *me*. Each of us is, but we're not the *it* that is. Or God, if you want to use that name.

And then you see the struggle of working that out. Plato directly deals with that. In Spinoza, he's the modern day Permenides. He says anything that is can *only* be in and through itself or in and through an other, and stress on *an other*.

Now, if the things we think exist, we identify them, all the ordinary things of every day life, and you, me, the cup on the table, that thing exists either in and of itself or in and through an other, by virtue of an other, and you have to cut clean on it. And what he does is he argues in that first book of *The Ethics*, it all has to go back to that *it*, the one being, use God as the name, that is in and of itself, and anything else that has its being has it through that.

Now, most people, or many will read Spinoza saying God or nature, God is nature. He identifies God with the world and everything. You read him more carefully in book one, he says God is that being that's a substance, an infinite substance as an infinite substance God has an infinity of attributes, each of which attribute is infinite, of which we only know two or three attributes—mind or thought, extension, and then I guess existence would be in there.

But it's clear he's saying but God has an infinity of attributes beyond our knowing. And then if you take Spinoza's explanation of knowing, human knowing, your knowing or mine, is perspectival; it's from a perspective, which means it's limited. And implied in Spinoza—and I think you could really carefully read *The Ethics* and show it, there is more than you or I, or all of us together can know.

Tegmark: How do you know that there isn't more to nature herself than you can I can know as well?

Balestra: Oh no, he would not deny that. But what he would deny, I think, is what Rebecca said. If you're going to say—

Goldstein: No, you didn't let me finish.

Balestra: Okay. All right. Sorry.

Goldstein: You totally, totally missed what I was trying to say about Spinoza.

Balestra: I'm sorry. Go ahead.

Goldstein: Certainly Spinoza's notion of both God and nature are completely different from what we think of as nature. When Einstein was asked if he believed in God and he said, "I believe in Spinoza's God" what he was saying—here is what Spinoza I believe says, not as metaphysical, and it's what Leibniz, his younger contemporary later called the principle of sufficient reason. It starts with a very simple question: Does every fact about the universe have an explanation? Is there always—and this is what Leibniz asserted is, yes, it does. It's a principle of reason that explanation goes all the way down. It's not turtles all the way down, it's explanations all the way down. There's no brute contingency in the world, so that there is always an explanation for everything, and ultimately what that leads to is that the world is necessary,

and necessarily exists. The world has to—or, the true, full conception of nature—which is not attainable to us, because we are limited to the two attributes, thinking and extension, so there's a kind of incompleteness theorem implicit in Spinoza's view. But the world itself, if we were to have the complete picture of it, image of it, which only God can have because it's infinite, it is in fact the infinite intellect of God, would explain itself. So transcendence, what he proves, if he's correct, is that the notion of a transcendent God is impossible. If in fact there is explanation all the way down, the world itself, the complete image of what nature is, if we had the final theory of everything, the final theory of everything would not only explain all the laws of nature, it would explain why these have to be the laws of nature. So when Stephen Hawking ends, you know, at the end of *A Brief History of Time*, he says, "Then we would know the mind of God," this is straight Spinoza talking. This is a real Spinoza. The full picture, the full mathematics, if you will, of the final theory of everything will explain itself. And that is I think what Spinoza is saying, and that is I guess kind of one of my only faiths is that it seems to me a reasonable assumption of rationality, that it's rational all the way down, that the universe always has answers, and if that's the case the universe does end up—we'll never get there—explaining itself. But in the notion of a transcendent God, that's the one that I completely reject.

Yes?

Levy: Is the *conatus*—I mean I always misinterpret this. Is that explained also?

Goldstein: Oh, the *conatus* is something in us. It is—so then he goes on, I mean one of the reasons that Spinoza was—and I don't want this to turn into a Spinoza—I know you already had a Spinoza forum—

Levy: Yes.

Goldstein: But—

Levy: And we liked it.

Goldstein: Yes. But, you know, he was vilified, excommunicated by his own Jewish community and then excoriated by greater Christian Europe, including Leibniz, who got some of his best ideas from him. He's a coward, Leibniz. But one of the things that he tried to do which was so heretical was to say not only do we need some sort of transcendent God to explain why is there something rather than nothing—one of the prime motives for a belief in a transcendent God—but we don't need God to derive ethics. His magnum opus is called *The Ethics*, and so he tries—he says that ultimately the answers for why is there something rather than nothing would come from a full picture of nature, and the answer of what is the difference between right and wrong and how are we to act and what should be the ends of human life comes ultimately from human nature. And that's what he attempts to do, and *conatus* is a fundamental view of what is it that motivates human nature. It is the urge to persist in one's own being and flourish—and from that he tries to derive all of ethics. So he took away from what he called the superstitious religions the explanation for why is there something rather than nothing, and what's the difference between right and wrong, and was hence considered a very, very dangerous man well into the Age of Enlightenment.

Tegmark: I think this is fascinating, actually. If you connect it back with what Edward Nelson said about how in mathematics you can have something that happened 2000 years ago and then it gets picked up again and new progress gets made, this very question, as you know, has come very much at the forefront of physics in the last few years, where we're in the middle of this very agonizing paradigm shift that makes most of my colleagues foam at the mouth about ultimately whether there will be these brute, unexplainable facts or not. So for example, Johannes Kepler, when he revolutionized the understanding of our solar system, he thought, he had this model where he tried to predict exactly the sizes of the orbits of the different planets, and he would probably be happy if we now today could predict the number eight from first principles; why are there eight planets in our solar system? Sorry, Pluto.

Levy: Pluto was excommunicated.

Tegmark: Yes. In company with Spinoza I guess. Of course now this sounds like a really ridiculous question, why are there eight planets, because the number eight doesn't tell us anything fundamental about our universe. It just tells us something about our address. We now know over 300 other solar systems. Some of them have eight planets, some of them have two planets, some of them have three. So eight, the number eight, if we had tried to have an explanation from first principle for where the eight comes from we would just have been tilting at windmills looking for something, an explanation for something which is fundamentally ridiculous to look for a fundamental explanation for.

And what's happening now is that that exact same demotion that happened to the number eight is happening to almost everything that we used to call basic laws of physics. We ask why are there one, two, three dimensions of space, why are there six quarks, rather than nine quarks or eight quarks? To the best of our understanding and the way it seems now, actually our universe is much larger than we thought. It goes on vastly beyond the part that we can see, and if you go sufficiently far away you'll come to a place where there are in fact eight quarks. So the number six also, just like the number eight, the number six for the quarks is just telling us about our address in this larger reality. So you look at these numbers, and in the end you realize this was just my telephone number. Eight wasn't something ultimately really fundamental.

So I don't know if Spinoza would be happy about this, in that it seems like maybe you can have a theory which requires no contingencies, or if he would be disappointed that this ultimate theory that we maybe will be finding actually is just telling us that ultimate physical reality is so big that all of the things we thought were facts were just part of our phone number or address.

Goldstein: But you as a sort of extreme Platonist seem implicitly to agree with Spinoza, I mean that the *big* picture, which is inaccessible to us—I mean, to me it's amazing that we know anything at all.

Tegmark: It's remarkable.

Goldstein: It's remarkable.

Tegmark: Yes.

Goldstein: I mean we're just these random products of evolution.

Tegmark: I mean we are. We evolved our brains to be able to pick bananas and throw rocks at each other [laughter], and here we are talking about Spinoza.

Goldstein: Exactly. But your intuition somehow also seems to be this big extreme Platonist picture, that ultimate reality is mathematics that explains itself. I mean it would seem your intuitions would be going that way.

Tegmark: I agree. If we look at the historical progression, we've again and again made the mistake as humans to assume that our universe was smaller than it really was. We underestimated the size of earth, we underestimated the size of our solar system, our galaxy and beyond, and much like ostriches, who put their heads in the sand, we figure that if you can't see it it just ain't there. So if you ask how far could this expansion of horizons go, what's the largest reality or multiverse that there possibly could be, and that this Platonic multiverse of all possible mathematical structures, all possible laws of physics as you might call it—it's sort of the extreme case that it could be. And I find that, if you think about it, first of all it's very dizzying of course, but second, I think it gives us some fresh air and sheds some new light on some ways of thinking about stuff.

Like you asked for example a very interesting question of the integers, were they created or have they always existed. That whole question presupposes the existence of time, because you need time for something to first not exist and then exist. A creation event requires time. But we know, and Einstein taught us that there are two ways in which we can think of time. We can think of either a reality being this three-dimensional place where stuff happens over time, or we can think of living in this four-dimensional space-time, a four-dimensional space where the fourth dimension has a minus sign in it, which makes it feel like the time. And the space-time of Einstein of course, there's nothing happening in there. If life is a movie then the space-time is like the entire DVD. It's all in there. The DVD isn't changing even though there's all sorts of drama unfolding in the play.

So if you think of our reality as a mathematical object which contains the space-time, then time exists within this mathematical structure, rather than the mathematical structure existing in time. So in the Platonic world I think nothing ever needs to be created. It would only need to be created if there was some sort of higher time—

Nelson: Of course not every physicist agrees with Einstein on this.

Tegmark: No, of course not. And I said I'm a minority of one probably among my colleagues.

Nelson: Arthur Stanley Eddington for example said that he believed that becoming is really there in the physical world, and I think he's right, as opposed to Einstein.

Tegmark: Whereas I would say if becoming is really there in our particular far corner of the Platonic world, where we live, where there is this time dimension and other mathematical objects, like the integers, which don't have any built in time in them at all.

Nelson: Oh sure, human creation, the story that human beings make up, that's my view of the integers.

Tegmark: So I have a great respect for formalism, and I think that's part of the power of mathematics that we can describe things purely abstractly, but my guess as to what's going on is that even though—suppose there's life somewhere else out there in space in a far away solar system, the beings who live there, integers might not be the first stuff that they start working on. They might study other formal systems and other structures. However, at some point they'll probably find it really useful to invent the integers, for whatever their goals are, and when they do they will invent—they will agree that five plus seven is twelve and discover exactly the same properties of the integers that we've discovered here. So I think different beings will be—they're all discovering the same mathematical, Platonic landscape, but they start at different places. But eventually with a sufficiently thorough explanation they'll probably find at least the same main boulevards and thoroughfares, all discover kind of the Broadway and the Times Square, even though historical accidents will determine that there are large parts that we never discovered over there in Brooklyn [laughter], and—

Nelson: It will be really interesting to see when we meet some alien intelligences. My own hunch—I have absolutely no evidence for it—is that their mathematics would seem very, very weird to us.

Levy: Wouldn't Kant, are you an opponent of Kant? I mean wouldn't Kant say that the integers were not invented, that they were—

Nelson: I've tried twice, once as a teenager and once as an adult, to read Kant, and I couldn't understand him either time, so I can't answer that.

Goldstein: Yes, Loren?

Graham: Yes, well I—as a historian of science I'm very much impressed with the fact that in the history of mathematics there have been crucial turning points. You know, imaginary numbers, irrational numbers, transfinite numbers, these were all extremely reluctantly accepted. There was a lot of opposition to them, and we could have gone another way. So I kind of guess, my hunch is that if we find mathematics in another civilization it will be, at least at first glance, quite different from ours.

But I just want to return for a moment to the more general theme of mathematics and religion, and science and religion. Most people who, or at least many people who raise this issue, the relationship between them, want to know do they favor each other or do they oppose each other, are they compatible or are they incompatible? And I'd like to make the case that that's the wrong question. I think you have to look at the context. If someone asks me to give examples of when science and religion fell into conflict, you know, it's easy: Galileo, Darwin, so forth. You know the stories. If someone asks me to tell stories about where science and religion favored each other it's easy: it's in Newton, it's in Pascal, it's in my name worshippers in the book I just gave.

On the other hand, if someone asks me to find examples of where atheism helped science, well I can come up with them. It's in early studies of *The Origin of Life* by JBS Haldane and by Aleksandr Oparin. It's in the development of Markov chains in mathematics, which was developed very specifically *against* a religious argument, based on Markov's atheism.

So, I would say that mathematicians and scientists get their ideas, their inspirations, particularly in moments of crisis, from all sorts of different places. It can come from philosophy, it can come from religion. It might even come from art. Those inspirations might come from atheism, it could come from all sorts of different places. So I think that we ask the wrong question if we do think that there's some kind of unique answer to whether or not science and mathematics and religion are compatible or incompatible. They're both. And there are many examples of each.

Nelson: I'd like to come back to sufficient reason. I've heard people maintain vigorously that the principle of sufficient reason is incompatible with quantum mechanics, there is not reason why this nucleus decayed at this instance instead of another.

Goldstein: Yes. Well, of course the response to those who hold to the principle of sufficient reason is that that is evidence that quantum mechanics is incomplete, and we have additional evidence that quantum mechanics is incomplete: it's incompatibility with relativity theory.

Nelson: The other possibility is that randomness and chance are a fundamental part of the universe we live in.

Goldstein: Yes. It could certainly be true.

Nelson: There seems to be evidence for that that's pretty strong.

Tegmark: And if Hugh Everett were here, who just down the road at Princeton postulated the many worlds interpretation of quantum mechanics, he would have given yet a third answer. He would have said we live in this larger space, this Hilbert space, and it did decay in some parts of this and it did not in others in his quantum multiverse, and, again, if I write down, say a bunch of zeros and ones which denote whether it did decay or didn't decay in successive experiments, that string of numbers is, again, just part of our address, part of our address now in this Hilbert space, and the ultimate reality is perfectly well defined. So that would be again an argument that there are no—if you call this a brute fact, it's just a brute fact in the same way that your zip code is a brute fact, but it's not that reality itself has anything arbitrary about it.

Goldstein: Yes. That's what philosophers call an indexical proposition, and that is, one could think that there's something—I'm a very special person, because I'm always here. Everybody else is always there. You know, look at me, I'm really very special. And of course that can easily be argued away just as an indexical proposition. It's the person who is stating it is always here, whoever that person is. Well, there might be something about randomness or something about the way things look because of our address in this particular part of the universe. And if you—actually, often people say what would Spinoza say about quantum mechanics, which seems to make randomness part of the universe, and I do think that Spinoza—it's hard to know what Spinoza would say [laughs]—but I think he'd quite like the multiverse. And there is a place where he says all possibilities are realized, and that's the multiverse basically.

Balestra: I'm not sure about that [laughter]. My sense is that here's a big difference between Spinoza and Leibniz, because Leibniz is the one who opens up the possible worlds and takes them very seriously. I don't think there's possible worlds at the end of the day in Spinoza, there are no *possible* worlds. All possibilities turn out to be the necessity.

Goldstein: Exactly. There are no possible—exactly. This world exists because it's the only possible world.

Balestra: Right.

Goldstein: Right, and so this world contains itself.

Balestra: That's very different than a world with possible alternatives.

Tegmark: On the other hand then, if this world is so large that it contains within it all possibilities, then it's sort of strange anticlimax, isn't it?

Goldstein: Yes.

Tegmark: Or, at least unification of the two viewpoints.

Goldstein: Yes, exactly.

Balestra: But I want to come back to something Max said and something you said, and I don't know that it squares. You made a clear distinction that mathematics, the language in—you know, whatever language that you use—mathematics is a priori knowledge. If you put it in that—now, is it analytic a priori or synthetic a priori? If you're a Platonist I guess you say synthetic in some sense.

Now, which means a priori means independent of sense evidence, sense experience, and yet it has a certain universality and necessity to it. And yet the physical world and knowledge of the physical world—and I don't want to limit myself to a narrow empiricist epistemology of that, of how we know the physical world is empirically, through sense evidence. At some point you need some kind of sense observation testing, but that's become very sophisticated today. No direct sense observation would suffice to test some of the theories of quantum physics or relativity—highly mediated by instrumentation and other layers of theory.

But here's what I wanted to say: I think, for Plato at least, in the end—and this comes out in his dialog called *The Timaeus*, about the creation, and this is what influenced Whitehead. This universe that is a becoming, that has spatial and temporal features to it—and what that means is the temporality is not a before or after. It can't be reduced to the before and after relation. It is a past, present, future, and it's a metaphysical claim. The past is something that no longer is, but somehow is really related to the present that is, and the present that is is somehow related to the future that will be. The way you talk sounded more like Spinoza. All that kind of in a sense gets engulfed in this one space-time structure with a mathematical thing. That's not a process universe. It's not a world of real creation becoming.

For Plato the physical universe is a becoming, because he talks about this—and you will never find in the physical world the perfectly embodied mathematical structure, because there's a third element to this world in space and time of particular things, of the physical, that resists full—whatever you want to call it—embodiment of the form, participation in the form. That's Plato's own language. What you might say today is full rationalization in a mathematical model. It approximates it, but it won't be fully and absolutely a mathematical structure.

Which is it you're saying? Is it clear what are the options?

Tegmark: My guess is that it fully is a mathematical structure, yes.

Balestra: Okay.

Tegmark: And I would also like to come back to the point that Loren made here moments ago about the basic question, the issue of on the one hand mathematics, on the other hand religion, and the question are they in conflict with each other or are they not. And you made this argument that in some sense it's the wrong question to ask, and I think a closely related question, which I find very interesting, is: if you look at people's responses to this stuff, if I look at my colleagues in the sciences and more broadly, I think they fall into three different categories. I think the first really major fault line is sort of if you talk about, well, Rebecca coined the ontological urge, this deep curiosity about the ultimate nature of reality. That's certain, as I said, what gets me up in the mornings and makes me so excited about my work. Some of my colleagues have it and some of them don't. And also outside of science, some people are really interested in these big questions—I'm guessing that the vast majority of those of you that are here are in that category—and some are just really uninterested in it and really find it a boring subject, and they're more focused on basic practical things.

And if you look within the category of people who are interested, really interested in these philosophical questions, I would say less than half of my physics colleagues, there's a strong sense of an ontological purge, if I may, in the physics community, where if people sense you're too interested in philosophy it's frowned upon. It's considered flaky. And for you it must sound very ironic that philosophical can be an insult [laughs], but in the physics community, for some reason—

Balestra: No, I understand it. I know—

Goldstein: That's actually how I went into philosophy, because I started out in physics and I was taking this course on quantum mechanics, and I kept asking my professor, "What does this mean? What does it mean?"

Tegmark: Yes, yes.

Goldstein: "How do I connect this up to reality?"

Tegmark: Exactly.

Goldstein: And he said, "Why don't you go talk to the philosophers." And I did, and I—yes, they were asking that question.

Tegmark: Yes, and often I'll say, "Just go off and do this homework," and then you'd be too tired to ask any more questions.

Goldstein: Yes, exactly. [laughter] Or their favorite response, "We don't ask that question. That question is meaningless."

Tegmark: Yes. And I think—my sense is that this anti-philosophical sentiment picked up in a vague way, particularly in America in the post war epoch. Maybe it was a little bit of a reaction to a lot of European idealist philosophies. I'm not sure, but for whatever reason it's still very persuasive. The P-word in physics is often leveled as an insult against people, and of course being a bit of a contrarian that just makes me even more determined to continue pursuing the questions I'm really excited about, which are these big questions.

Goldstein: I think it's probably a leftover from logical positivism that was so dominant.

Tegmark: Maybe so but—

Nersessian: Would it be the same thing in psychoanalysis, besides philosophy.

Tegmark: Yes.

Graham: But wouldn't you admit that all the crises moments in physics, that when physicists really faced a kind of intellectual abyss and they were trying to get over it, and they knew they were going to have to think in a new way, that at those moments they go out and read philosophy.

Tegmark: Absolutely.

Graham: Einstein read Mach, Heisenberg read Kierkegaard, Heisenberg read Plato—

Tegmark: Absolutely.

Graham: At moments of crisis they do get philosophical. It's just in the every day that they don't.

Tegmark: And even the ones who claim to be completely uninterested in philosophy tend to actually have their own poorly articulated philosophy anyway, and they use their own clunky terms for standard notions in philosophy, because they couldn't be bothered reading the books. So I have little patience for that, but if you just accept that there is this big dividing line anyway, first of all those who have the ontological urge and those who prefer the ontological purge, who just are not interested, then I think people who have the ontological urge can naturally be very interested both in fundamental science and in religion. I think that's something they really have in common, because ultimately—you know, clearly religion has been very much about trying to understand the big questions, which you only do if you're interested. Our degrees are doctor of, not physics, but philosophy in the very name, you know, echoing these roots.

And then if you look within the group of people who have the ontological urge I think there's a second very important division, which is those who are tolerant and those who are not. [laughs] In other words, we sit here with a very diverse bunch of beliefs and views going into this, and we're fascinated to hear viewpoints of each other, and we're not throwing water at each other yet. [laughter] Whereas of course there are quite a few people in the science community, and particularly in the more fundamentalist areas of human thought who just are not interested in understanding other people's point of view. And I feel, as a scientist I feel I often have much more in common with people who are tolerant and have the ontological urge but believe very

different things from myself. I have much more common ground with them and a much more interesting time discussing with them than with any kind of intolerant people, or with scientists who don't have the urge.

Goldstein: Loren, I wanted to—oh, I'm sorry—

Nelson: I would like to return to something you raised a long time ago, Rebecca. You said that when you were a graduate student and studying Gödel's work this led you towards Platonism against your wishes.

Goldstein: Yes.

Nelson: Well, perhaps there's a way out.

Goldstein: Tell me, please.

Nelson: Gödel's second theorem says that if arithmetic is consistent—*if* arithmetic is consistent then the consistency of arithmetic cannot be truth in arithmetic. Now, the reaction of the mathematical community to this is rather strange in a way. It was universally accepted as just a fault of the weakness of arithmetic that it couldn't prove this theorem—which everyone knows is true, so how do you prove it? You prove it in set theory, very simple proof. Well, how do you know set theory is consistent? Well, you prove it with large cardinals. It's like a defense attorney, and he notices his client is not going over very well with the jury, so he brings in a defense witness, and then he notices the defense witness is arousing suspicion, the character witness is arousing suspicion of the jury, so he brings in a character witness to the character witness, each one more Mafioso than all the preceding ones, and where does that get to? Nowhere. So perhaps the trouble, the obstacle was that you cannot prove the consistency of arithmetic in arithmetic, if arithmetic is consistent, an important proviso, perhaps the obstacle is that arithmetic is inconsistent.

Goldstein: Well, that's a fine kettle of fish. I mean from an inconsistent system you can prove anything.

Nelson: It doesn't link to Platonism.

Goldstein: Sure doesn't. And—

Tegmark: But if it is inconsistent then there is a finite linked proof that zero equals one.

Nelson: Yes.

Goldstein: Exactly.

Nelson: From the axioms of arithmetic, which include the induction axioms. That's where the suspicion lies.

Tegmark: So in principle you can prove that it's inconsistent. So if you find that proof let me know.

Nelson: Okay, I will.

Goldstein: Well, I mean an inconsistent system has a very, very serious flaw, because from an inconsistency anything can be proved, so in fact—

Nelson: Right. So it'd have to be altered.

Goldstein: Yes.

Nelson: The induction axioms arouse my suspicion very strongly. They're based on the a priori notion that the natural numbers are complete at infinity, and I think the notion of complete at infinity is a human fabrication which has unsuspected logical difficulties with it.

Balestra: That I think connects with the lecture this morning about is there an actual infinity, and you're saying no.

Nelson: No.

Balestra: And if there isn't then is Cantor's work then in trouble?

Nelson: Yes.

Balestra: Okay.

Goldstein: Oh, but it's so beautiful. How can you say that?

Nelson: What is beautiful in mathematics is proof, and well, Cantor is a bit vague, but certainly there are beautiful proofs in set theory. Those are of permanent value, whether or not the axioms are consistent.

Balestra: You know, it's interesting, I'm listening to this and I'm thinking Spinoza, Leibniz, and then some others. Descartes is interesting here. On mathematics he was for his time brilliant, I think much more brilliant a mathematician than Galileo. And you know the Galileo question, you alluded to it, that he said, but he didn't show it or explain it, that the world is a mathematical structure. Descartes argued for that metaphysically, and it's a brilliant attempt—I'm not saying it's right, but it's brilliant. And cunning in that Hegelian sense, the cunning of reason.

But what is interesting, in his argument for the existence of God, in *Meditation III*, he's very careful, after he does it he anticipates some possible objections, but what he says is—you know, he argues from the idea of this infinite being, to the existence of the infinite being. It's not Anselm's argument, the classical ontological argument. It's a really new, unique one. But in arguing it he kind of says, "And don't think I mean by this idea of infinity here the mathematical infinity"—you know, that series of numbers. He says, "I could explain that from my mind. I can generate that unending series." And he never says it's actual or not an actual infinity. He just says, "What we mean by that is generating those unending series." He says, "That's not what I mean by the infinite when I speak of God." So he makes a clear distinction, since we're talking about mathematics, and now in this case God, the concept of God. Descartes says, "That's not what I'm talking about." And then he claims that he has this idea of God as infinite being, and he

makes this distinction, and this is where—you know, it's a tough question. Does he have an adequate idea of God as infinite being? And he, in response to Arnauld, who was a contemporary philosopher, brilliant guy too, he says, "The fact that I have the idea—it's adequate for me to make the claim I have the idea if I can somehow comprehend it, even if it's incomplete, but not fully grasp it, hold it completely." And that's a fudge. But I think what he's raising is this key notion, is there this idea of the infinite. And I think that's something Spinoza thinks he's got. He's got this insight into the idea of the infinite, you know, the whole. But even there Spinoza admits, "I know enough of it as infinite so I recognize it has an infinity of attributes of which I only know"—and then he accounts for why he only knows those, you know, under thought and extension and existence.

Graham: What you say about Descartes points out to me a real difference between religion and mysticism. There are quite a few outstanding mathematicians who have been in one way or another mystics. Descartes wasn't one of them. He was outstanding, but he wasn't a mystic. A mystic believes that knowledge comes directly through some path other than logic or rationality. And so a mystic would be totally uninterested in Descartes effort to prove the existence of God through logical argument; that wouldn't touch him.

Balestra: No, I understand what you're saying.

Goldstein: Loren, I wanted to ask you a question. You had—and then perhaps we should open it up to the audience, but I just wanted to ask you a question. You had alluded to the arguments, the mathematical arguments you said that you take seriously, and I'm not sure what they are. So one of them is, look, maybe a lot of mathematicians—if it turns out that a lot of mathematicians are creative because of their religious beliefs—and actually, I had read this poll and they polled these various disciplines, scientific disciplines about are you an atheist or not, and so it was kind of what are your theist or lack thereof commitments, all the way from theist to deist to agnostic to atheist. So which group do you think came out as the most atheist? It was economists. [laughter]

Graham: The dismal science.

Goldstein: The dismal science. Then it was biologists. The more theoretical the physicist, the more they described themselves, whatever that means, as religious, and the mathematicians were pretty high up there. Well, what if it turns out, just psychologically, that a lot of mathematicians do have this—and I think one could even come up with an argument why mathematicians might be disposed—they spend their life in the abstract. They're often not very interested in the material world. If you've ever spent a lot of time with mathematicians you know this to be the case. You know, so, okay, I can see that orientation. But if that turns out to be the case, that mathematicians do tend toward religiosity, I mean, one, all you would have here is a correlation, which might have a common cause, this kind of non-empirical orientation towards the world. You don't even have causality here that it's the religiosity that's causing their mathematical brilliance. A lot of mathematicians, very great ones, were also mad. They spent a lot of time in asylums, including my guy Gödel. So this kind of orientation is no argument for—even if it turned out to be true that a lot of mathematicians were religious, I don't think we would get any argument for religion out of mathematics that way.

Or you could be mounting a different kind of argument that, look, our knowledge of mathematics, especially if you're a Platonist, is just so mysterious. What could have given us, we packets of slime that we are [laughter], what could have given us this access to this realm? It must be the divine spark in us, you know, that sort of divine madness that Hermann Weyl spoke about.

So, which of those two arguments, or neither, are you referring to?

Graham: I would need a lot more evidence than I've seen so far to come to the conclusion that mathematicians are in some way uniquely religious. I'm not foreclosing it. I'm just saying I haven't seen the evidence that would make me think that. To me that's a very different argument from the argument about whether or not some very important things in mathematic developments have had religious or anti-religious dimensions. They have. And that can all be illustrated. But I'm not going to yet accept the view that mathematicians are unusually religious.

And on this question that you—very interesting question on which the economists came out as being the least moved by religious considerations, you know, every poll-taker knows that everything depends upon how the question is asked and what the definitions were. One poll doesn't show much here.

Goldstein: Well, why don't we open it up to the audience?

Levy: You have to come up to the mike if you want to—

Audience: My question is directed to Professor Tegmark, and has to do with something Professor Goldberg said—

Levy: Goldstein—

Audience: Goldstein, I'm sorry.

Goldstein: That's okay. I answer to any Jewish name.

Audience: Professor Goldstein, if I'm remembering what you said correctly, did you say that monotheism presupposes that it could have been otherwise, it's contingent, and if I'm remember correctly what you said, Professor Tegmark, I'd be very curious what you have to say about that in light of your theories of levels of multiverses. And in particular the question with respect to this concept that Professor Balestra brought up of the presupposing that there was a past and presupposing that the future is based on this, when that's not true in some multiverses. There is more than one time dimension. So if you would address yourself to that, please.

Tegmark: Yes, so my guess is that if you assume—if you buy my presupposition that everything which can happen according to some mathematical laws of physics really does happen somewhere, then anything which you might describe as contingent, could take place this way or that way, will definitely happen somewhere.

Audience: Thank you.

Audience: I don't know who to address this to, maybe everyone. I feel like asking you. The term infinity has popped up an awful lot today, at your talk and here, and I was kind of sitting back there and wondering, because I've run into this in a couple of cases. When are we going to—mathematics has changed over the years. There are new rules that pop up every now and then. We've mentioned some of the names of the people who formulated them. When do you think we should start to accept the fact that we have a new technology, namely the computer, to possibly show us the way into another way of going—let me give you specifically what I mean: I think the last number that I heard for a twin prime pair, or perhaps it's just a single prime number, is something on the order of ten to the sixteenth, and you mentioned that classic problem of forming even numbers I believe. When are we going to accept the fact that, well, okay, we've gone, let's say in the first example to the ten to the sixteenth, we've identified something, and there's no doubt that one day that number will be ten to the thirtieth. So when are we going to accept the fact that—or higher. When are we going to accept and allow that proof is obtained without the usual analytic procedures that we're used to classically?

Nelson: Never, I hope.

Audience: That may be, but why?

Nelson: Because, speaking—I think I can speak for the vast majority of mathematicians here: because it's ugly. Mathematics is a search for beauty, and to prove that something holds for all numbers, no matter how far out you go. That's what gives us joy. Knowing that it holds up to ten to the thirtieth, well, that's only a slight way up. What about ten to the ten to the ten to the ten to the ten to the thirtieth? Why be satisfied?

Tegmark: This leads straight to the question my father has often asked me, which is: Suppose mathematics really is inconsistent, and there is a proof that zero equals one, but the shortest proof has more steps in it than there are particles in our whole universe. Then does it really matter?

Audience: Thanks for helping me out.

Nelson: How would one know that that's the case?

Audience: Thank you.

Audience: Yes, on the question on number of scientists or mathematicians who are religious or not, there are studies going back a hundred years. The last one was published in *Nature* in the late 1990s, and basically the higher up you go, I mean the more acclaimed the scientists are, the fewer believe. And the average is only about 10% of all scientists—

Graham: What do you mean the higher up you go?

Audience: If you're in the National Academy of Sciences or you won the Nobel Prize, those were—well, a hundred years ago they took a different category, but that's the basis of the last study. And in the last study, the average is 10%, but biologists, only about 7% believe, and for mathematicians there's about 15%, which is still a very, very small number, but it's double what it is for a biologist. So since you asked about why is that so, and the only reason I've been able to find out is that there are studies done with twins, and there appears to be a belief gene that when

they take two twins who are separated at birth, if one was put into a religious home and one was in a nonreligious home, either they both became religious or they both became nonreligious. So that's how they—you know, they tend to defy the environment, so there could be some genetic factor.

Goldstein: Maybe I was adopted.

Audience: May I say that this is a remarkable panel. I drove down from Boston to attend. I was stunned by reading some of Professor Nelson's essays. I fit into a number of categories you've discussed. First of all, I'm Russian, I'm Orthodox, I'm a Christian, I'm a mathematician—rather applied mathematician, a student of a phenomenologist, the late Gian-Carlo Rota. I hope that he will forgive me whatever—I will not try to characterize his responses. I wish he were sitting there. I'm sure he would have had much to say.

But if you will allow me I will duplicate that Norwegian Cruise Line commercial recently that just overflows with words [laughter], and I've written them down here. First of all, in relation to your question about math and science vis-à-vis religion, you asked that question, but then you talked about not math and science, but mathematicians and scientists, the people. And also—so, in that sense I would suggest we should perhaps stick to some, not necessarily Platonistic version of the content, rather than the contentors.

Secondly, when we talk about religion, as an Orthodox Christian, I'm not a fundamentalist. And I think much of the, for example the PBS series on evolution put up a straw man, put up a Baptist fundamentalist, they're easily blown away. The origin, if you will, in the Middle East, before 1054 was not fundamentalist. And it is more mystical, has a lot of aspects that you've touched on. And so I beg you to consider the fact that religion is not literalist, it's concrete. I still believe in the scripture. On the third day it says that the plants came forth with their seed inside themselves, and yet the sun was created on the fourth day. That's not a problem for me as a believer. I can use it as a hypothesis, along with the late Fred Hoyle, that life came from somewhere else, not necessarily from our solar system. So I can use the scriptures as a very nice scientific hypothesis.

If I can just have your patience for just a little bit more, you were discussing the infinites. You may not know that in 1300 there was a terrible controversy between a group of monks called the Hesychasts and Rome, and it was a violent thing much like we have today in our public life, and the issue was the Hesychasts in their inner contemplation claimed that they saw the fire, the energy, the spark of the divine. And of course this is outrageous, both in Judaic and in Christian terms, because we can't even say the name of Jehovah, and also to presume something in the Christian world as well, that we're talking to God in the Whitehouse, this is also a big no-no. And yet there came forth a saint, Gregory of Palamas, who formulated a resolution to all this. He called it apophatic theology; we define God in terms of infinite—you know, *in*comprehensible, in a lot of negatives, so how are we to know him? And this goes back to the stunning words in Edward Nelson's essay. He says he believes in the *in*carnation. So there's a lot of—I beg you to not be deflected by false straw men or fundamentalists and literalist minds.

Coming back, one more minute if you will. I'm also a student of gravitation, and I wanted to address Dr. Tegmark here, because I've seen you in that wonderful video made by *what's his*

name [laughter], you know, where he interviews everybody—including Alan Guth—in the whole world of cosmology. But I wanted to say that a lot of physics today is very peculiar, you know, multidimensional, and we don't have 95% of the universe tangible. But in part this is due—and also, as Freeman Dyson said in a wonderful paper called *Missed Opportunities*, he said we don't have a compatibility between quantum mechanics and relativity—gravitational relativity. Well, the irony is that the man who invented special relativity blew it when he went general, and the whole world today is grounded in that conventional theory, which has no exact two-body solution, by the way, and the man that I have worked with has a wonderful response to this, fundamentally principled on special relativity, from which he derives a metric and then the field equations—

Audience: Thank you, I'm done—which has no black holes, no weirdness, no inversion of time and space inside the horizon and so forth. So anyway, I'm done. I want to thank you. Karen Armstrong is entirely wrong when she says—and I stopped reading her *God* at that point, when she said science supplants the need for God. I just stopped reading.

You are wonderfully philosophically educated, and I thank you.

Audience: I had a thought, but I'm going to add another one, since he talked a lot. I'm Greek Orthodox, I'm not fundamentalist. I also have two science degrees. I'm also an actor and a writer, so I'm both left and right brained. And I think there's probably a multidimensional way to look at everything where everything could fit together, like evolution could be true, Adam and Eve could be true in a way that we can't think it or express it yet. It doesn't have to always be either/or.

And another thought that I had here is a good future discussion might be with musicians—I sing a little, play a little guitar, so I'm not a composer or a pianist or a violinist, but I hear there's a lot of math in that, and I'm sure when they're all into their Zen of being at Lincoln Center let's say, they feel something like divine spark, the Holy Spirit, however they want to put it, and that would be also an interesting discussion, since that's mathematics and art, and maybe religion, to them put together.

Levy: Thank you.

Audience: My question regards formalism and Platonism. It's always seemed to me as if they were more one, and I was glad to hear you say that David Hilbert was a Platonist. And this is the question: When a formalist writes a set of axioms—you know, say David Hilbert writing his axioms for geometry in the 1890s—isn't that person describing a mathematical object, a Platonic object? And then it goes on to be a mechanical process, but the original writing of the axioms and designing the systems, isn't that a Platonic act? That's my question.

Goldstein: I think, Ed, you should respond to that.

Nelson: Well, but did the object exist before the mathematician started writing about it? I've seen no evidence for that.

Balestra: Whereas I do.

Audience: At that point?

Balestra: Yes. I think that's Platonism, to say if you hold that—in some sense that object exists before the mathematician started writing or thinking about it, and it is a discovery when he finds his axioms, the descriptive axioms hold up. They work. Now, that's a different interesting question: Well, how do we know that? Then you have to do the proofs. And I guess from a mathematical point of view, unless you derive a contradiction, you can't. And that's what makes Gödel's proof so powerful, because you see there are going to be some theorems that are in principle unproveable, and so you kind of say, well, now we have to make a philosophical move that's beyond proof, so to speak, and a certain conviction, and you live with it and see where it takes you.

Audience: It's the way it is.

Balestra: When you say *it's* the way it is what's the way referring to?

Audience: It's the way reality is. It doesn't have a proof or explanation. It simply is.

Balestra: Right, but I'm not—see, your question, are we agreeing that the reality you're referring to is that mathematical object in regard to its being there independent of having a proof or not—but then how do you know it? That's where the knowledge question comes in. And to the extent that the mathematician or the physicist comes up with the mathematics and seems to be on to something, then, yes, you know it. But to say—because my sense is, Max, you want to—that strong, Platonic conviction of his says if there are right now these unresolved problems, like with quantum physics and general relativity theory, in time they're going to be embedded, resolved, almost in Hegelian fashion, and in a new kind of higher level mathematics that encompasses that and will give us ways to solve that. Is that the kind of thing you—

Tegmark: Yes, and I think the way you put it resonates very much with me, when you say there is this external reality that exists independent of us humans, this external physical reality, and it *is*. It exists, it is, it doesn't require any words of the English language to do its business. It just is, and then we can try to describe it, and if we want to describe it without any human language, then mathematics, this purely abstract formalist language, is the language that we need to use to describe it.

Audience: Yes, my question really touches on Max's concept of the mathematical universe, that you can't leave out the cosmological constants, that our universe is precisely defined by certain numbers, which to me, I wonder how that touches on God.

And the other one is the notion, in terms of divine inspiration to do mathematics, is the mind of the autistic savant, which calculates numbers and all sorts of mathematical relationships without any interpersonal skills or any awareness of the universe.

Tegmark: Yes. So you're referring to this really striking fact, that the more we've studied nature the more fragile, or fine-tuned we've discovered it is. Every single number we have ever been able to measure in science we can compute from a list of thirty-two numbers, pure numbers, pure mathematical numbers without any physics units on them. And we've discovered if you tweak most of those numbers, even by just a percent back and forth, the whole universe gets completely

messed up. The sun explodes or there are no oxygen atoms or there are no carbon atoms. And the three basic reactions people have had to this, those who find—those who are for the ontological purge say brute fact, suck it up, move on, think about something else. [laughter] And then there are people who argue that this fine-tuning is evidence of some formal creator who twiddled these knobs to make the universe suitable for life, because that seemed like a kind and nice thing to do. And then there's the third school, which is that if we really live in a much larger reality, there are theories which predict that those constants take different values in different places. And then of course we expect to define this as living in one of those few bio-friendly habitats where the conditions are just right.

Audience: I was intrigued by this problem of making one equal zero, and it occurred to me that perhaps the question is zero and infinity are somehow really interconnected. And when you talk about multiverses you're avoiding the problem of a set of universes being *the* universe, which is all the *is*. And the idea of creative emptiness, the emptiness of possibility, which is a Buddhist concept, I think it's very close to some of the stuff you're working on. I just liked your thoughts.

Tegmark: My thoughts about infinity and physics are that we're really in quite a—a very embarrassing crisis in physics at the moment, because there were all sorts of issues we can't—which you've heard of, like as the failure to unify gravity with quantum physics and so on, which have to do with infinities cropping up left, right and center. And yet, even though it's very convenient to talk about infinities as an approximation in their physical theory, we've never seen anything infinite as physicists, neither the infinitely big nor the infinitely small, which is implicit in the assumption of the continuum that we can measure the distance between two points within infinite decimals. We've never measured anything in physics ever to more than fifteen decimal places, okay? And that's far short of infinite. And I'm still—my guess, which is, again, very contrarian among my colleagues, is that ultimately maybe there are no infinities in physics and this is just a convenient approximation we make, because it's easier to do the math this way, and that we'll discover that the laws of physics that we actually have are somehow, there's something finite under it all, which kind of looks at—much like the air in this room feels smooth and continuous, even though we actually know that we're just inhaling a bunch of discrete atoms.

Audience: I'm a physician, an MD, and I was just thinking about PET scans, like when somebody has a stroke, or when a certain man had a stroke, like part of has brain was ablated so he couldn't do math, so eventually different channels in the brain, or different neuronal connections came so that he could relearn math, but with different circuitry. So I'm just wondering, like you said that you started in mathematics and they said go talk to the philosophers, if everybody did a PET scan on the four speaker's brains, I'm sure they would light up in different areas, like the mathematicians would light up, I forget which part of the brain, or their homunculus would be more homunculi here [laughter]—and there really could never be—not never, I shouldn't use that word. But just the interplay of religion and God depends on like what your neurocircuitry is. Like, for me mathematics, like I was mandated to take calculus. To this day I hate it, and I've never used calculus ever; a real and unreal number never walked into my office demanding primary care. [laughter] But the whole discussion is interesting. The point that I'm making is that we're all so different, and I think that you said that it takes everybody so that we could just advance our thinking, which will never be a certain thinking, but uncertainty is really good. It may be uncomfortable, like we'd like to know is there an afterlife, but I don't think we'll ever get the answer until we're there, so—

Goldstein: That's a very good point that you're making, and I want to address it slightly, and that is: I think one of the things that we're capable somehow, we packets of slime [laughs], of experiencing something that, it's sublime, you know, that kind of lifts us right out of ourselves, and it's one of the most intense and wondrous experiences that one can have in life. I mean it has something to do with working very, very intensely, and you lose, you know, you kind of lose sight of yourself and you kind of come back into yourself and time stops, and one can have—the different kinds of temperaments and minds that there are—I mean first of all I think it's probably some small subset of people who are capable of this kind of intensity, where they do get lifted out of themselves, but maybe not such a small subset. And some of us get it in math, some of us get it in theoretical physics, some of us get it in music. I think art in general is a way for many people to experience this sense of sort of the sublime and being lifted right outside of yourself, and I think we often use religious language to—you know, because when you come back into yourself you come back a little differently, having been out. Your whole view of things changes. You know, you realize what a petty little creature you are. And you could—the religious language exists to describe it, and it's hard to describe it in secular terms, and so I think that might be one of the reasons why—you know, Hermann Weyl would say, you know, it's the spark of the divine. He experiences an enormous, this experience of sublimity doing mathematics or doing theoretical physics. Others of us experience it in other ways—creativity does it in general. And so, you know, it seems to me that that's not a very good argument for religion, but it shows the kind of proximity there. We use religious language often to describe this experience of the sublime.

Audience: Mathematics seems to be making fewer problems for us in the social world than religion does. What do you make of that? [laughter]

Balestra: Yes, I want to say not for mathematicians. [laughter] Religion makes very little problem for mathematicians, and mathematics makes *many* problems for mathematicians.

Levy: One more question.

Audience: I have a question to Loren Graham, and the question is like this, and it has to do with your book: When you talk about Florensky, his, I guess, philosophy of mathematical entities, if I'm not mistaken you write that Florensky thought they were human creation. Is that correct? Or is that the NKVD thought that Florensky thought that they were human creation—

Graham: Unfortunately both.

Audience: Both. The question then is like this—I mean I read Florensky on contour, and Florensky seems to think that, you know, set theory is directly about God. I mean at least that is the sense that I have reading it, that God is the object described by it. How does one go from numbers, or from mathematical entities are human creation to that move that describes God? Isn't it easier to do from Platonism?

Graham: It's a wonderful question, but it also fits with how did Florensky think that he could prove that there is a God? His answer was, "I know that God exists because I name him." So Florensky was creating both God and mathematics.